SOME NETWORK CONCEPTS OF BOARD GAMES

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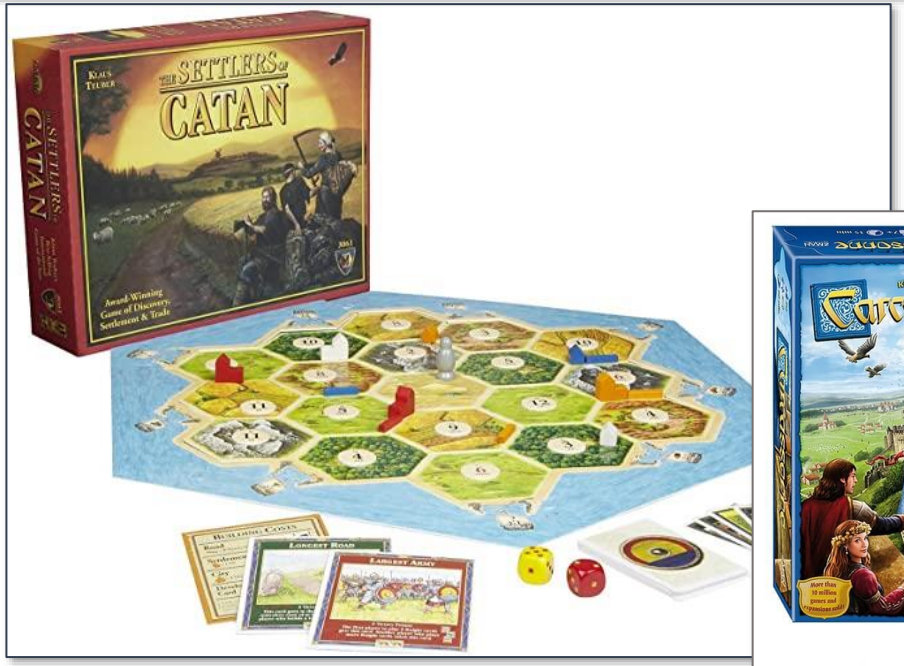
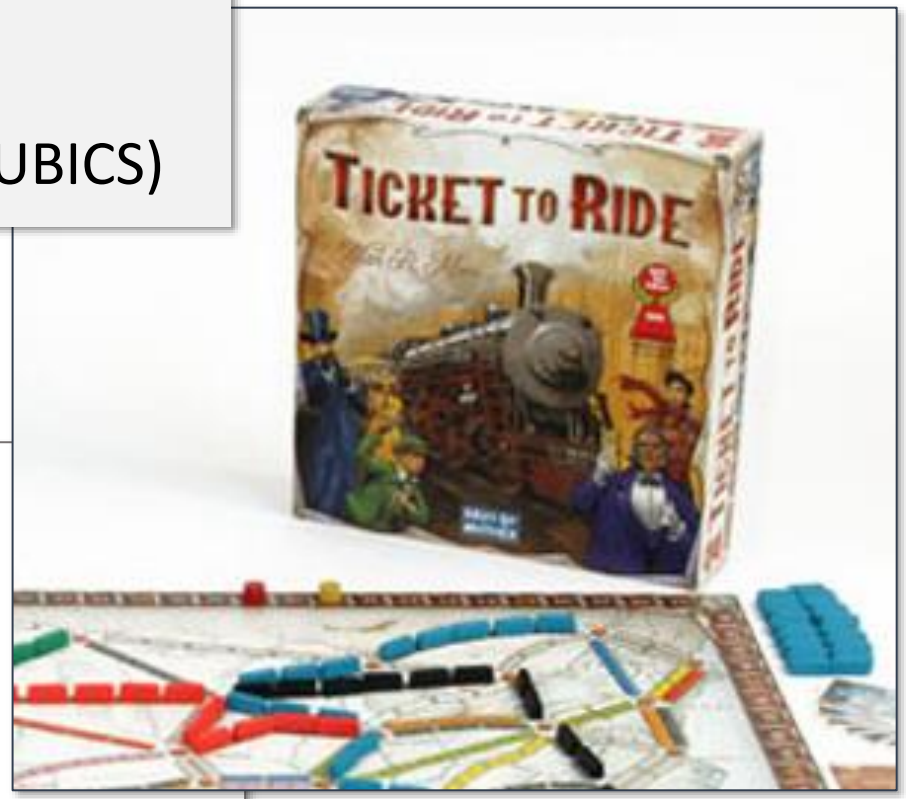




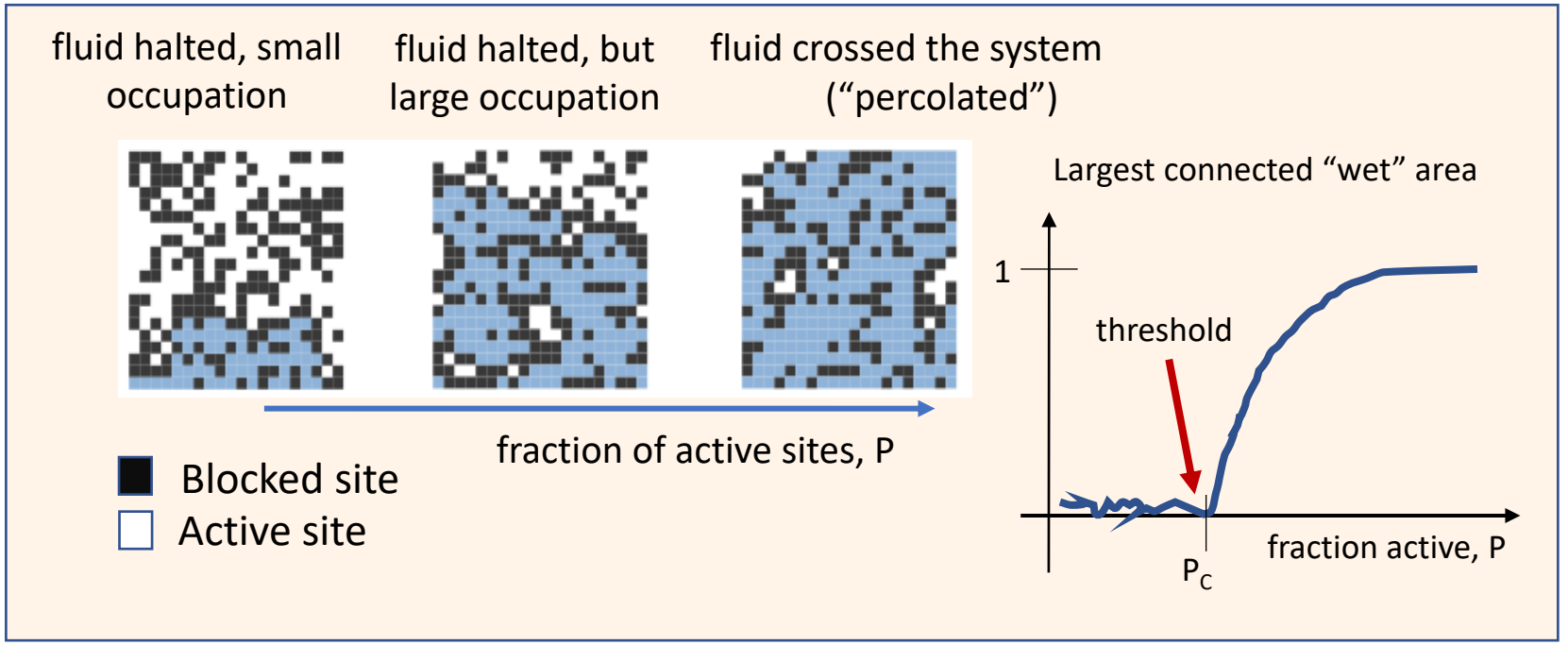
TICKET TO RIDE & CATAN:
Percolation in complex networks & giant component

Percolation

- Percolation is a classical problem in Statistical Physics.

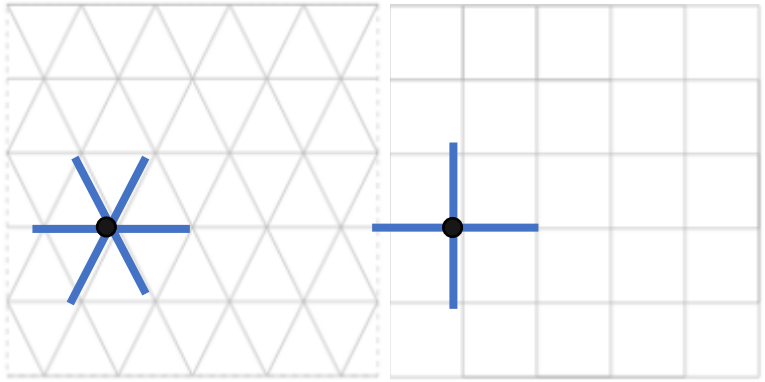
System of interconnected sites that can have two states, **active** or **inactive**. Active sites allow the flow of some observable (information, a fluid…) across the system, while inactive ones block it.

What minimum fraction of sites P_C must be active for the observable to cross the entire system? How much observable is present as the probability of occupation increases?



Percolation

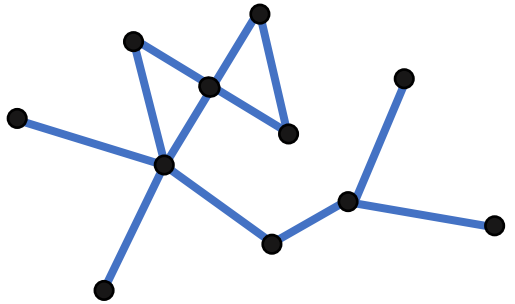
- **Bond percolation**.



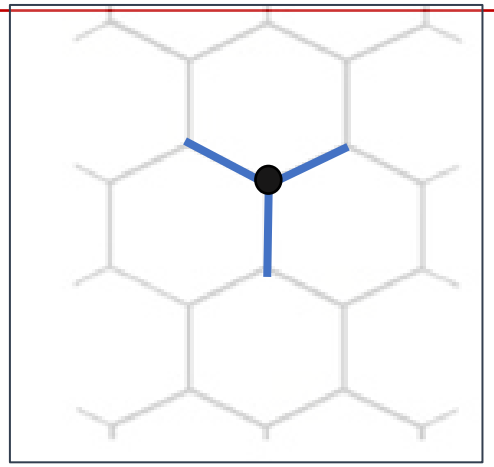
6 links per node
(constant K)

4 links per node
(constant K)

General network



(varying connectivity per node,
one should considerer <K>)



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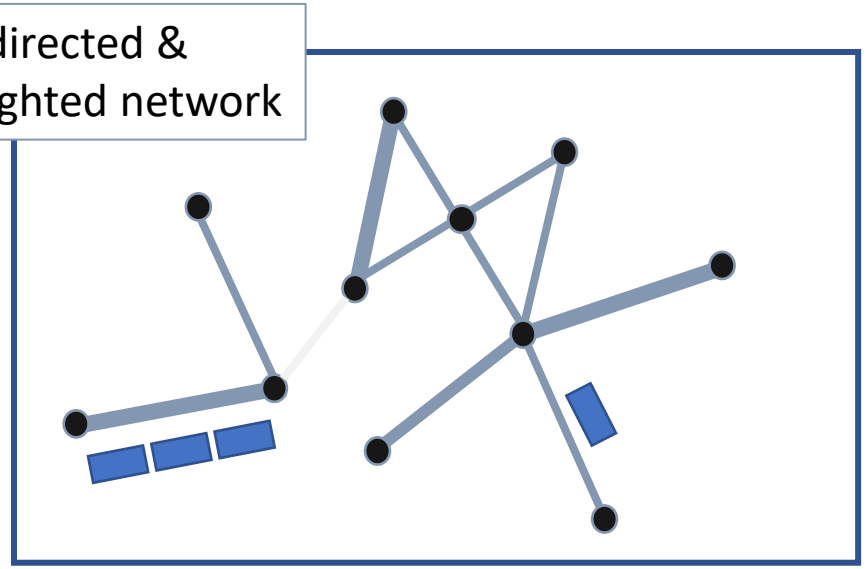
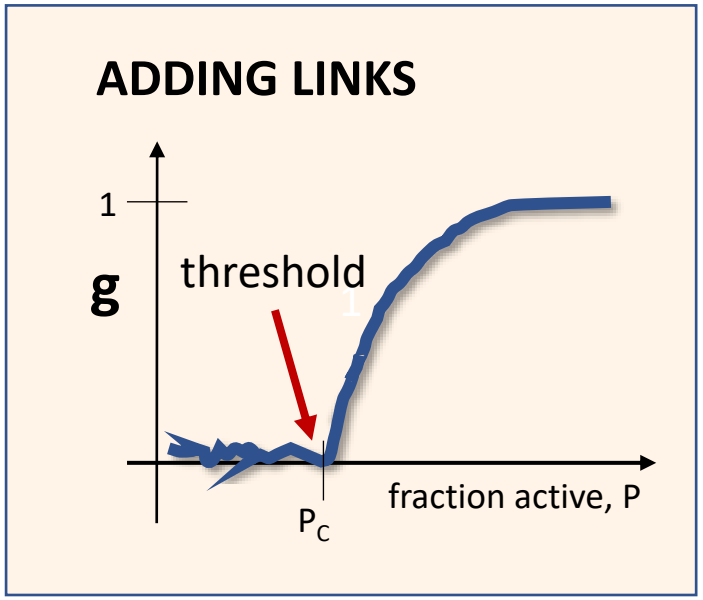
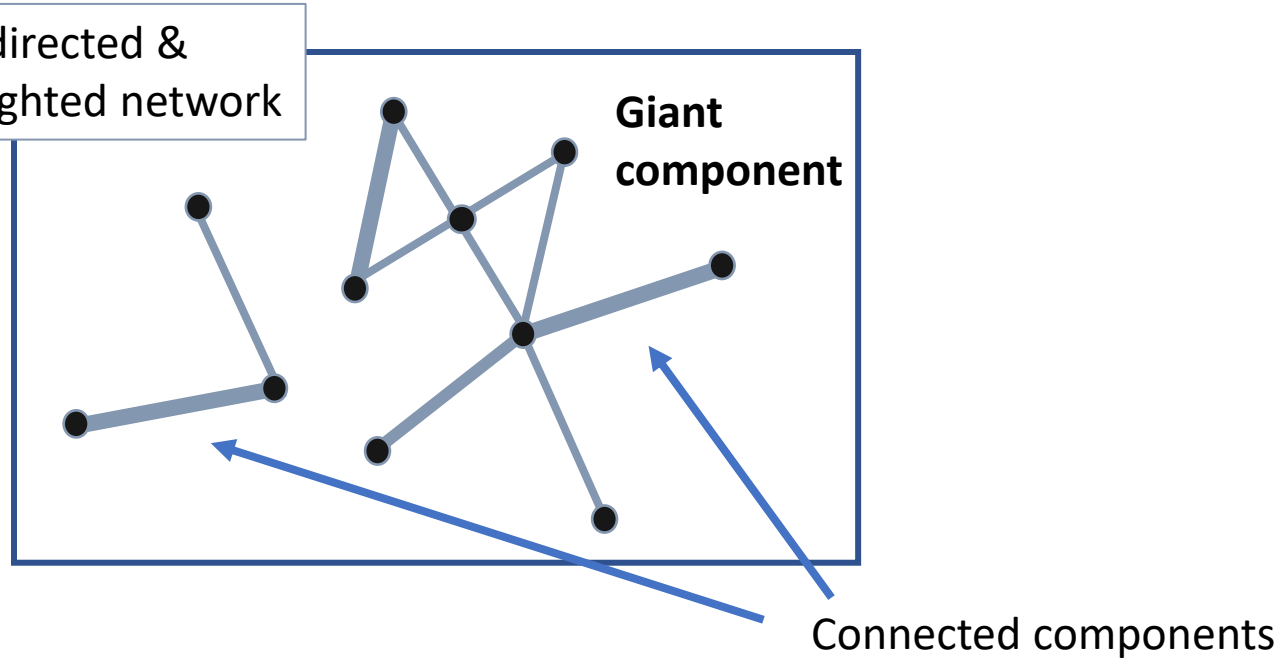


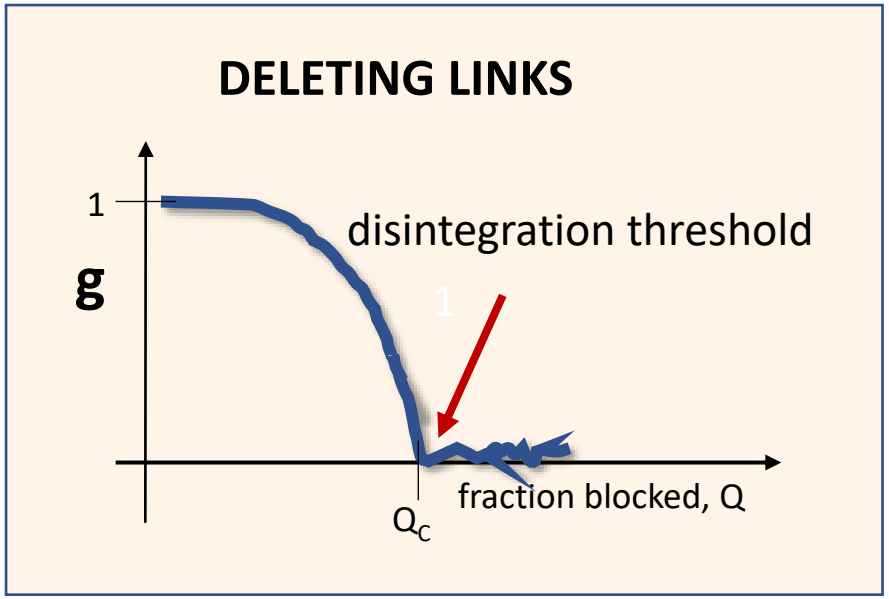
Percolation

■ **Bond percolation** and its importance.

- The giant component g is the number of connected nodes. g=1 implies all nodes connected at least through 1 link.







In Ticket to Ride:

- Get points for connected components.
- Get points for 'percolating' (long routes)
- Get points for giant component.

Percolation

shape depends on Network structure

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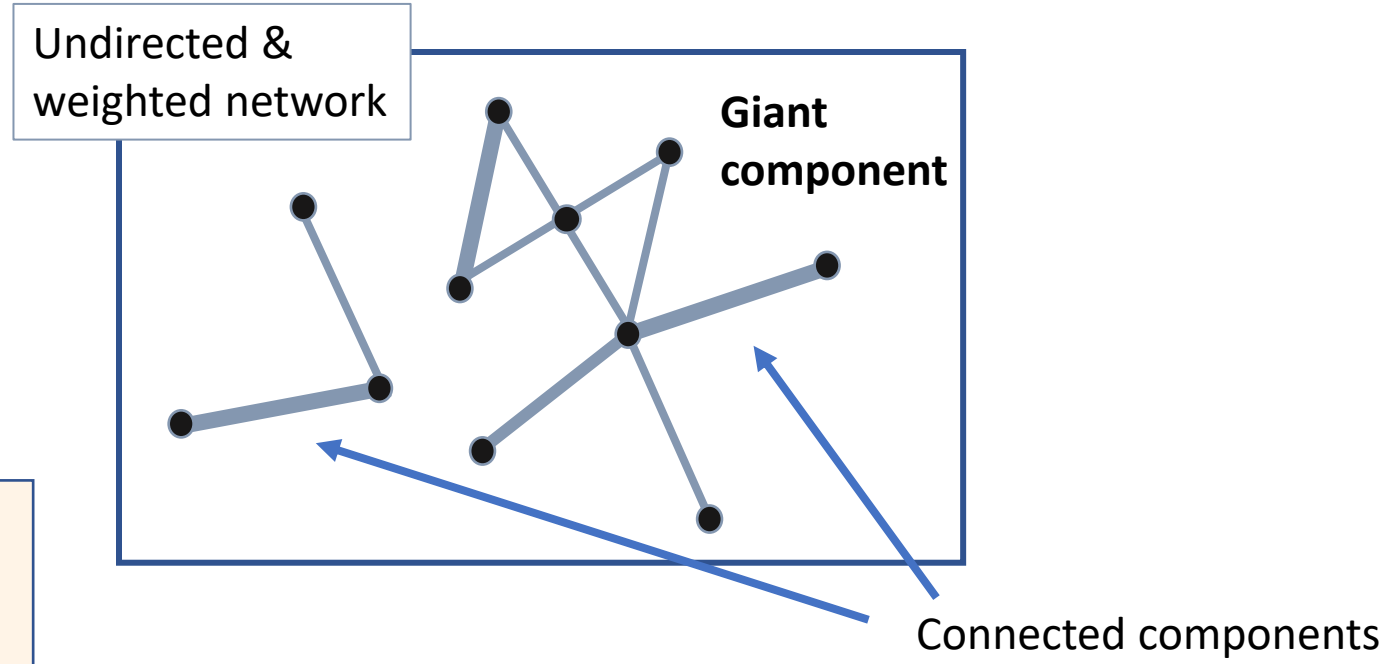

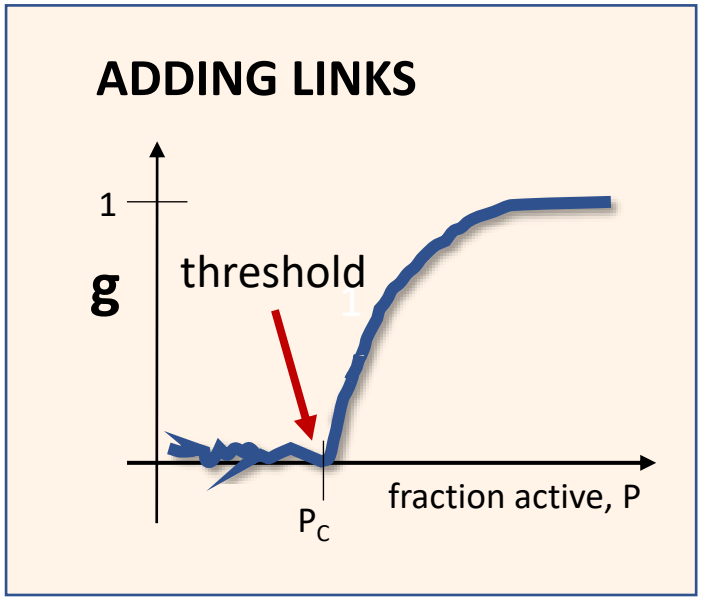

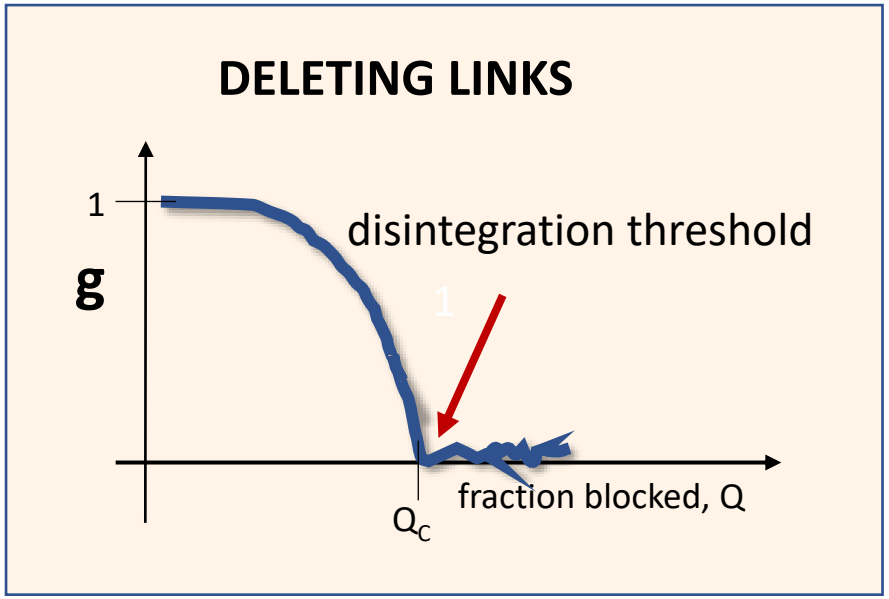

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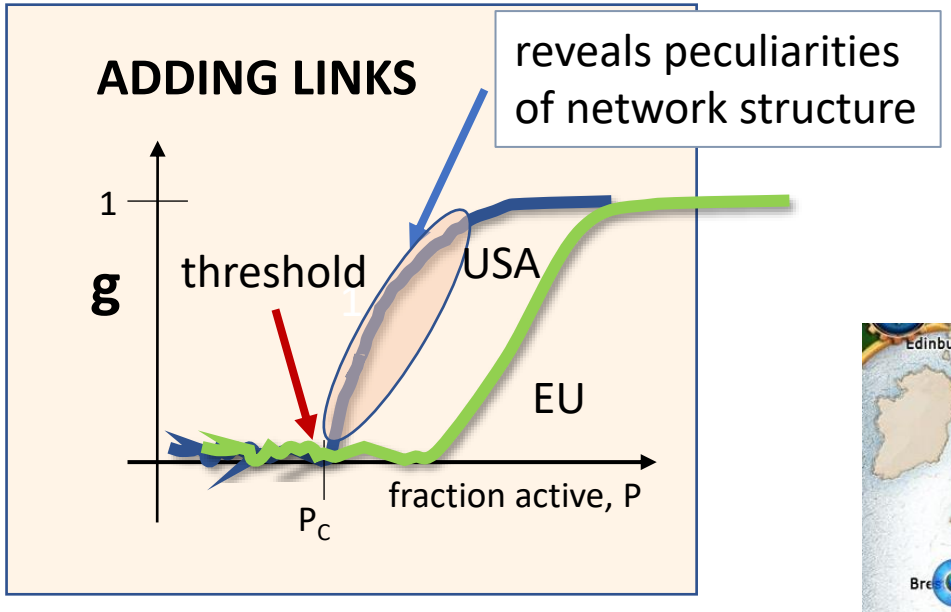
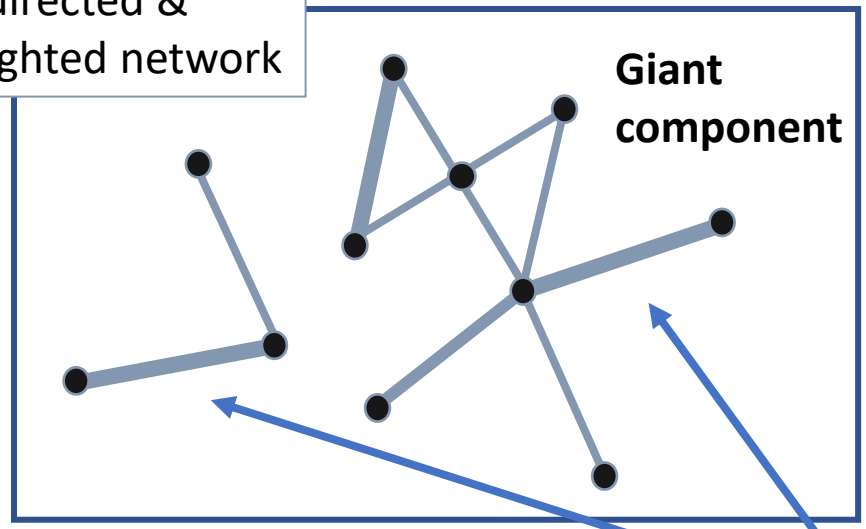
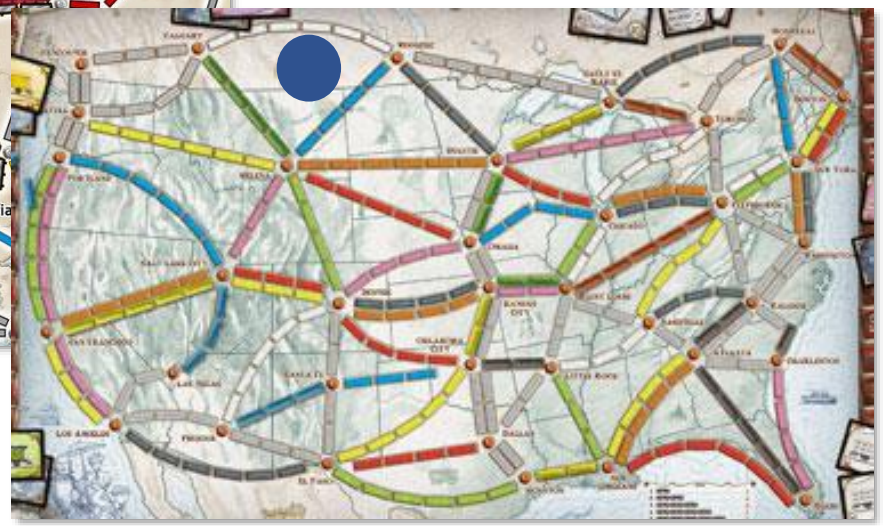






Other features:

- Efficiency: EU map needs more links to be filled to cross it completely.
- Spatial networks: EU geography constrains connectivity.
- And if the think in players (colors), we have a very rich multiplex network (color=information traveling).

Percolation

A physical model for Ticket to Ride?

ARTICLE

https://doi.org/10.1038/s41467-018-08009-9 OPEN

Bond percolation in coloured and multiplex networks

Ivan Kryven [1]

Nature Communications 2019

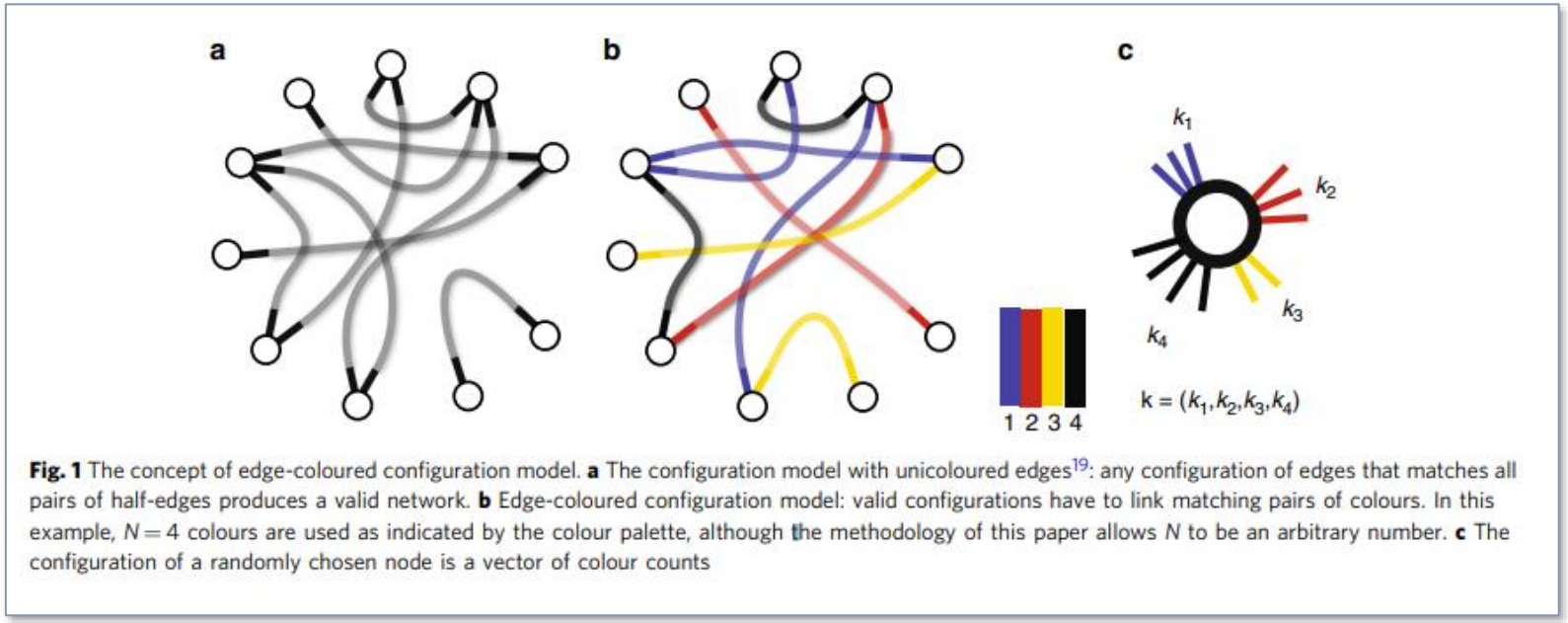
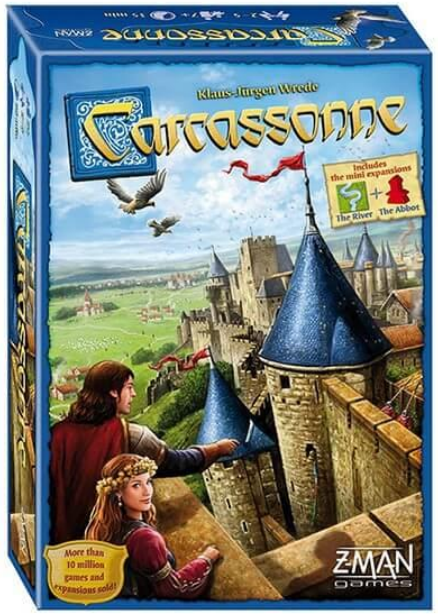


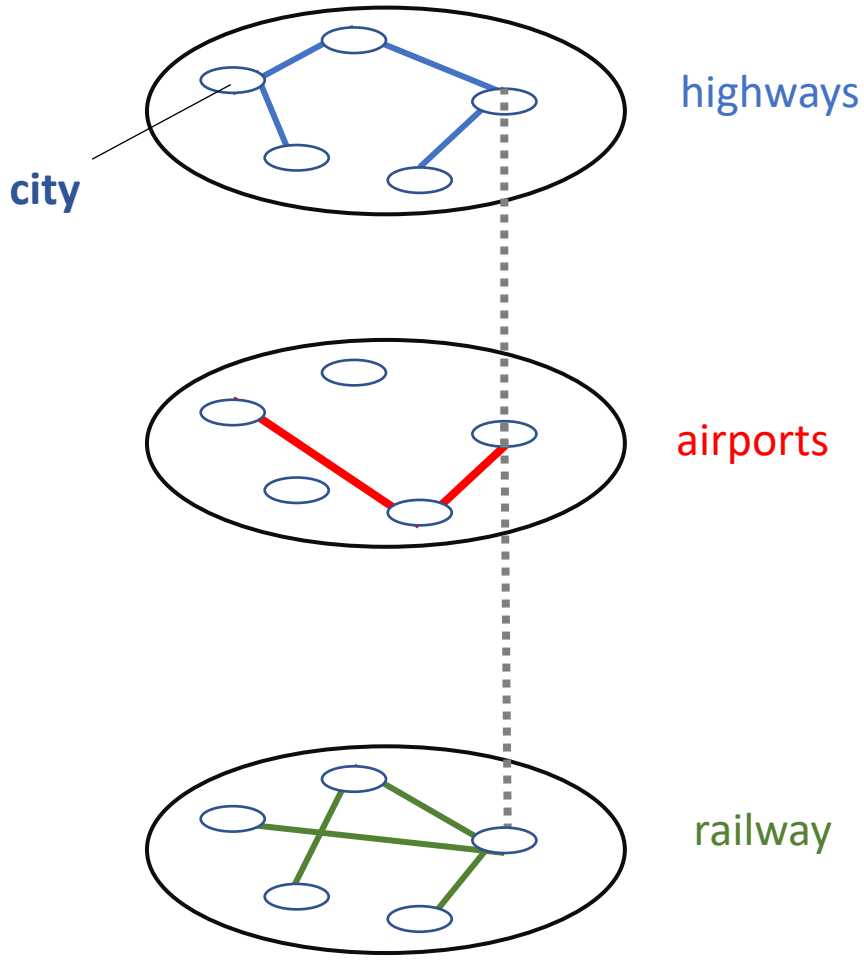
Fig. 1 The concept of edge-coloured configuration model. **a** The configuration model with unicoloured edges[19]: any configuration of edges that matches all pairs of half-edges produces a valid network. **b** Edge-coloured configuration model: valid configurations have to link matching pairs of colours. In this example, $N = 4$ colours are used as indicated by the colour palette, although the methodology of this paper allows N to be an arbitrary number. **c** The configuration of a randomly chosen node is a vector of colour counts

CARCASSONNE:
Multiplex (or multilayer) networks

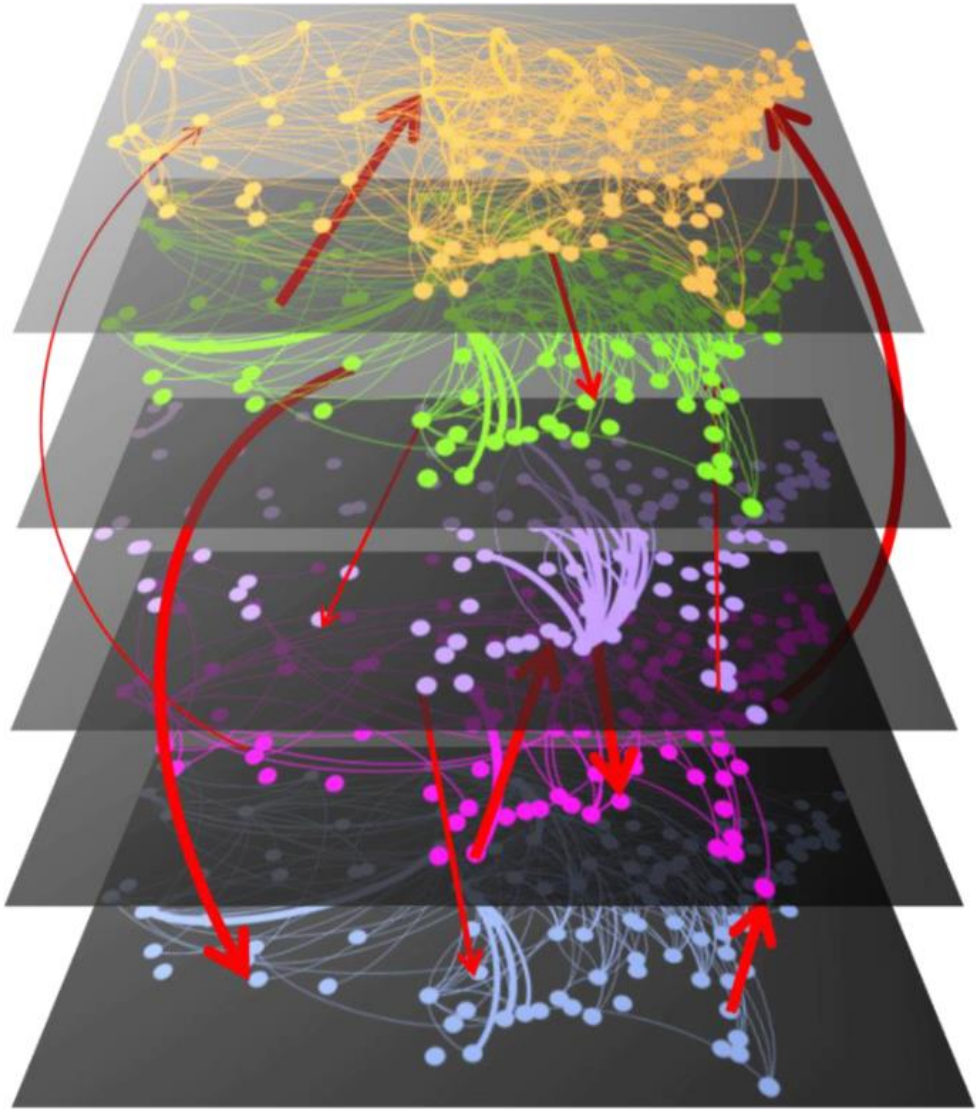


(SPATIAL) MULTIPLEX NETWORKS

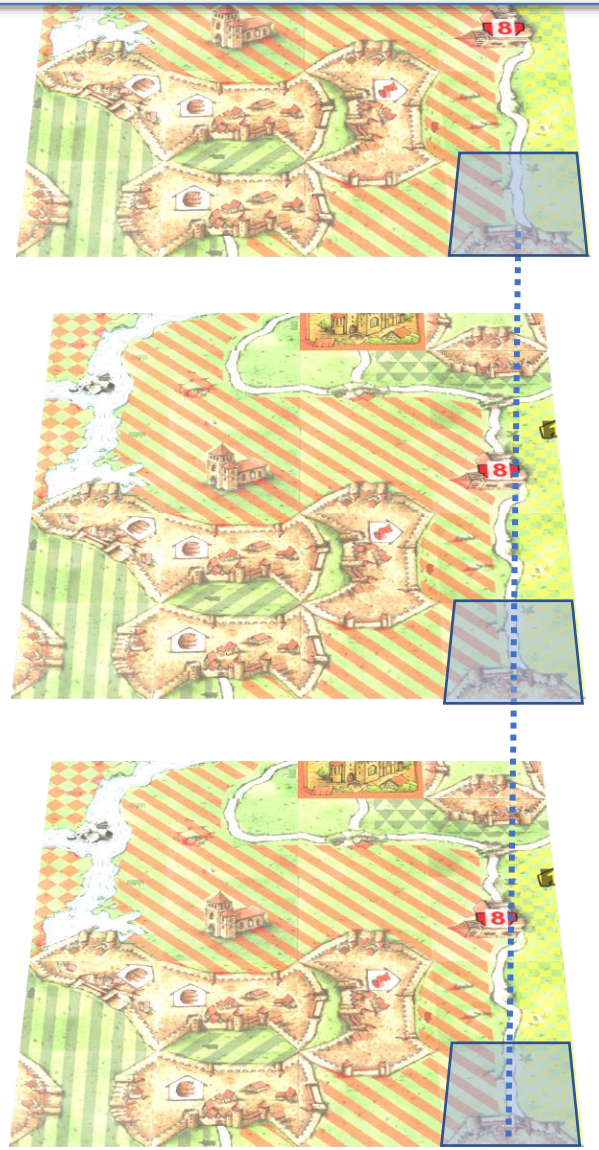
COUNTRY: a complex set of interconnected cities

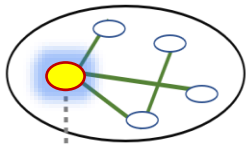


Example: fragility of supply chains in the US
[transport + logistics (internet) + production/demand]

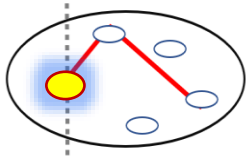


In Carcassonne, a tile represents 3 different concepts, shaping a multiplex network of 3 layers (or 4 layers if cloisters are treated differently than cities).

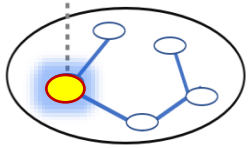




FIELDS



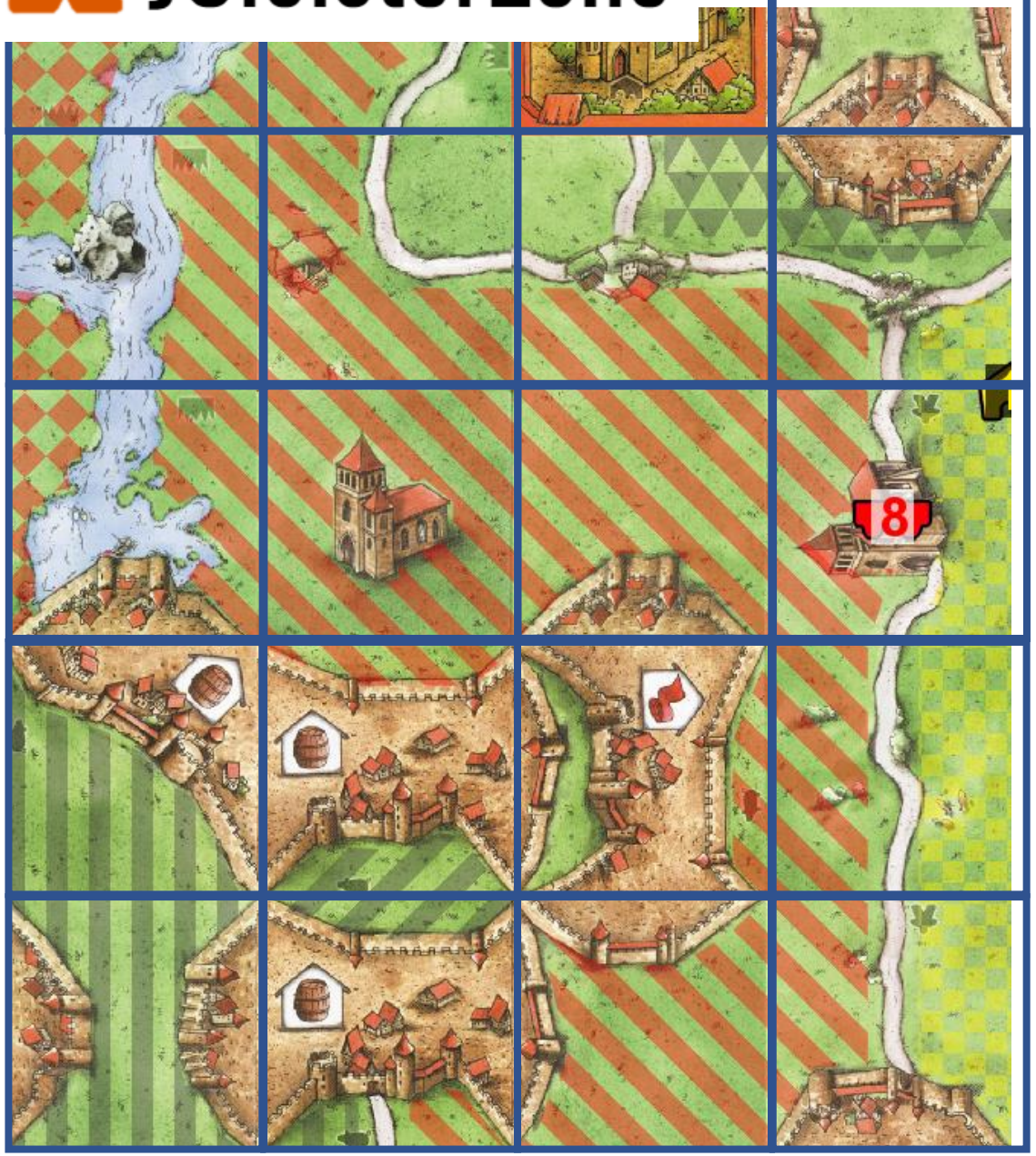
CITIES



ROADS

https://jcloisterzone.com/en/

JCloisterZone



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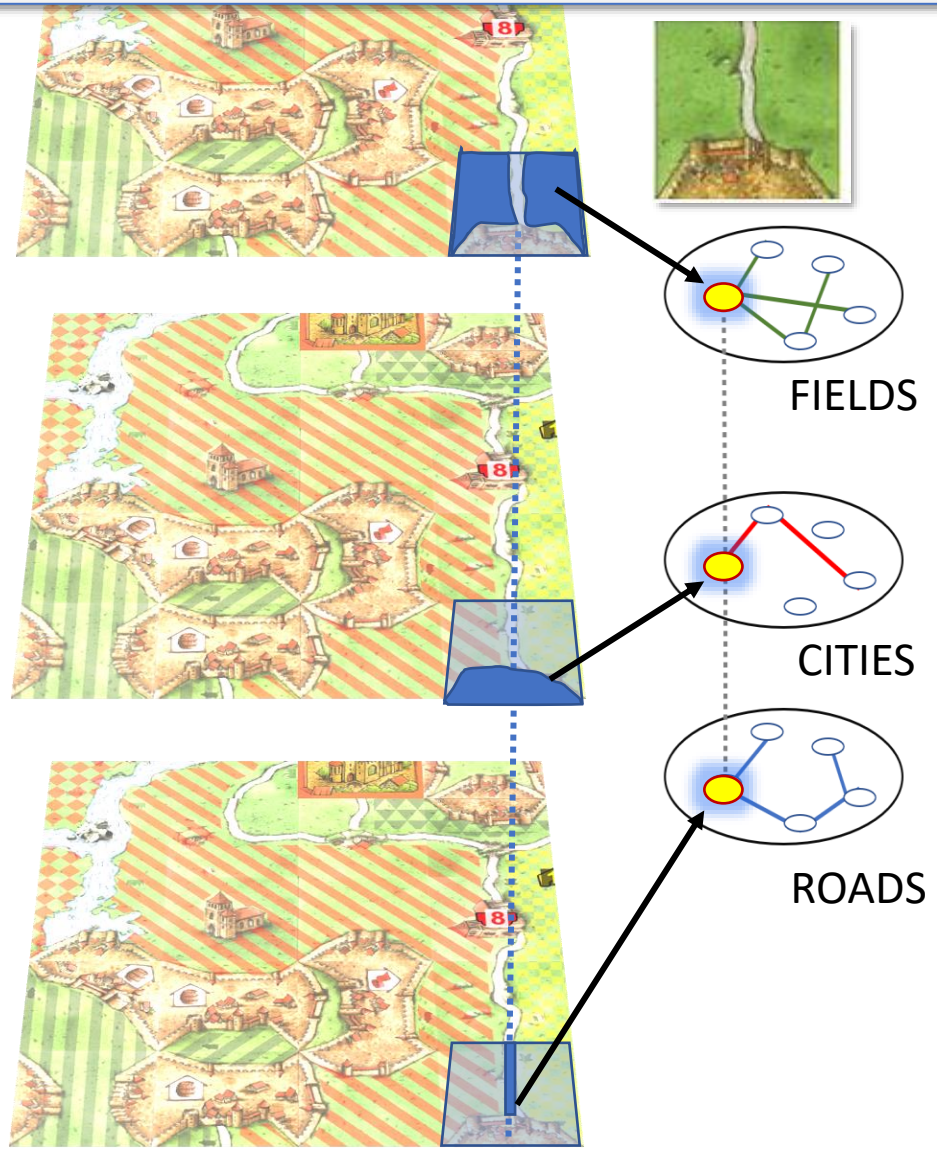
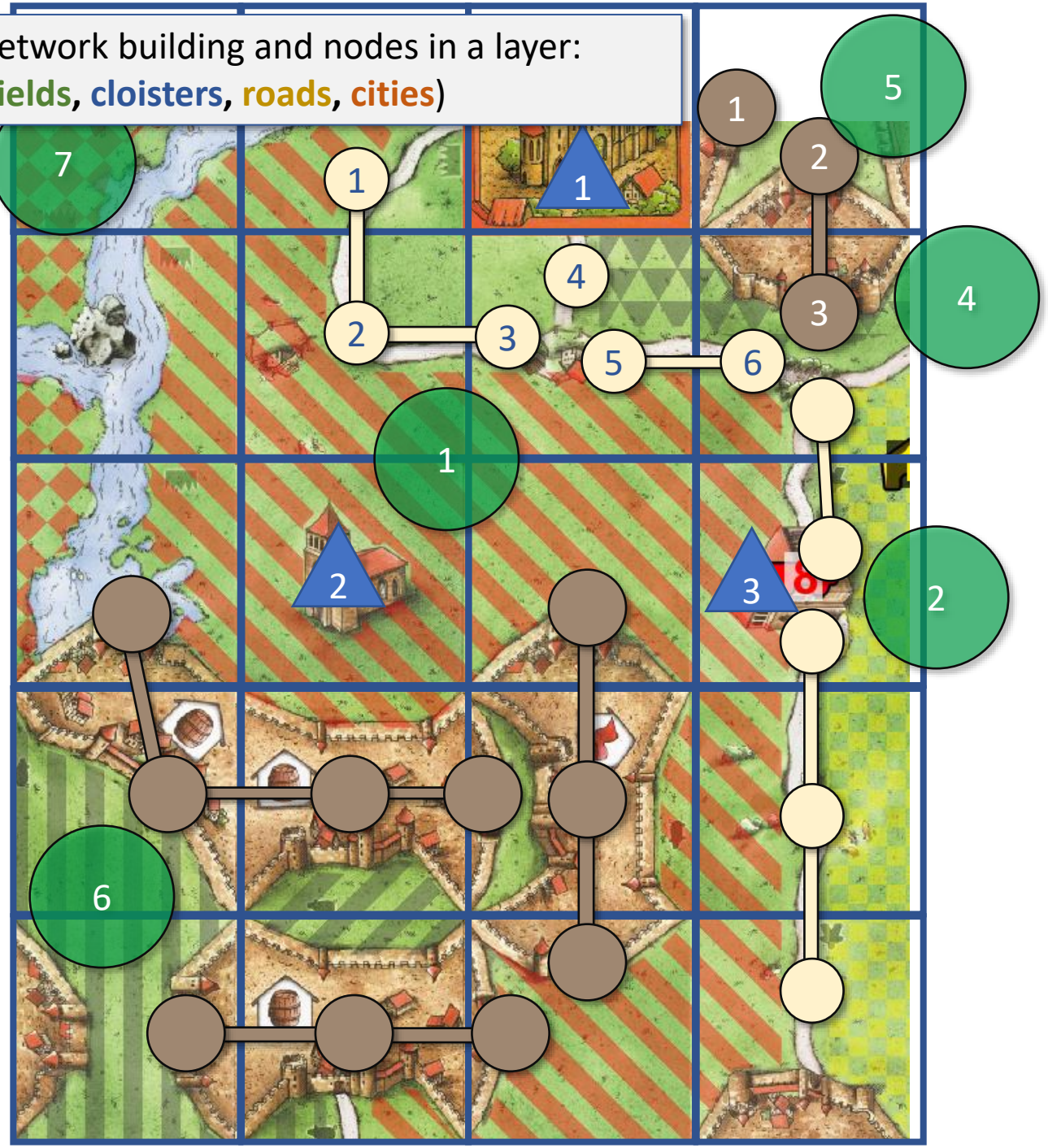
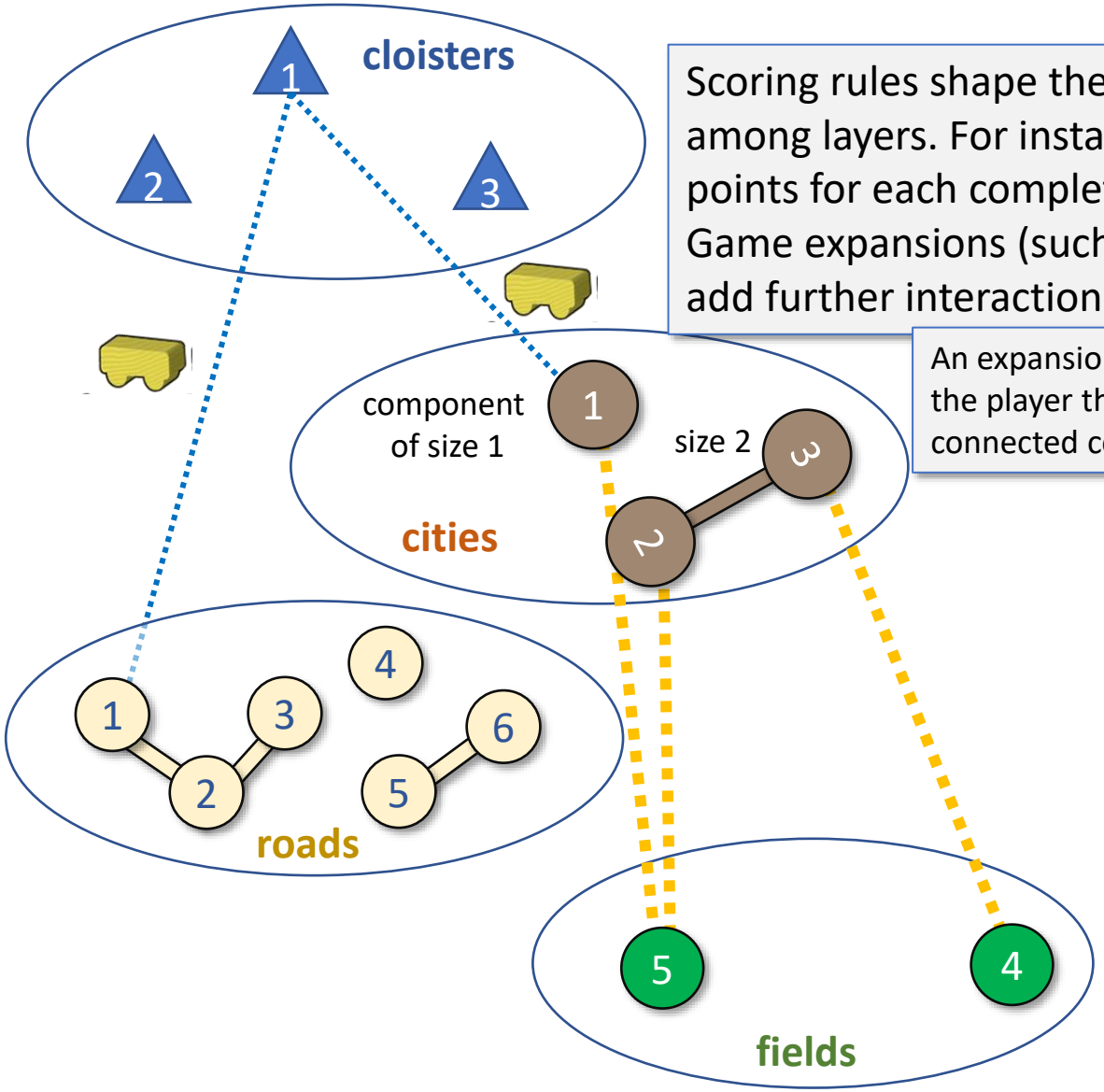


https://jcloisterzone.com/en/

Network building and nodes in a layer:
(**fields**, **cloisters**, **roads**, **cities**)



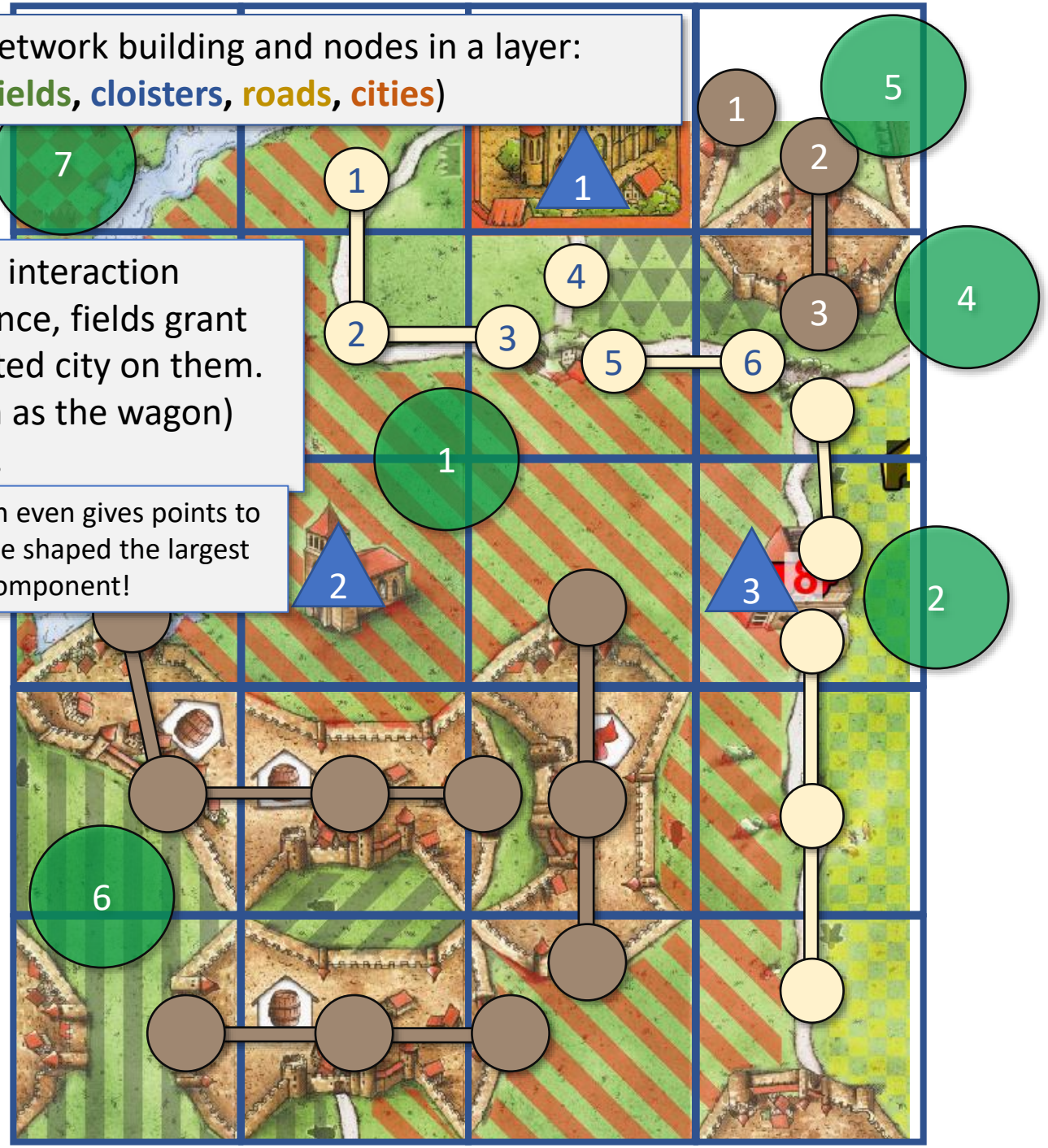
At the end of the game, each layer contains a set of components of different sizes. The player that built the biggest components normally win.



Scoring rules shape the interaction among layers. For instance, fields grant points for each completed city on them. Game expansions (such as the wagon) add further interaction.

An expansion even gives points to the player the shaped the largest connected component!

Network building and nodes in a layer: (**fields**, **cloisters**, **roads**, **cities**)



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THE EUROPEAN PHYSICAL JOURNAL B

Colloquium

Towards real-world complexity: an introduction to multiplex networks

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Abstract. Many real-world complex systems are best modeled by multiplex networks of interacting network layers. The multiplex network study is one of the newest and hottest themes in the statistical physics of complex networks. Pioneering studies have proven that the multiplexity has broad impact on the system's structure and function. In this Colloquium paper, we present an organized review of the growing body of current literature on multiplex networks by categorizing existing studies broadly according to the type of layer coupling in the problem. Major recent advances in the field are surveyed and some outstanding open challenges and future perspectives will be proposed.

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Multilayer networks

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In most natural and engineered systems, a set of entities interact with each other in complicated patterns that can encompass multiple types of relationships, change in time and include other types of complications. Such systems include multiple subsystems and layers of connectivity, and it is important to take such 'multilayer' features into account to try to improve our understanding of complex systems. Consequently, it is necessary to generalize 'traditional' network theory by developing (and validating) a framework and associated tools to study multilayer systems in a comprehensive fashion. The origins of such efforts date back several decades and arose in multiple disciplines, and now the study of multilayer networks has become one of the most important directions in network science. In this paper, we discuss the history of multilayer networks (and related concepts) and review the exploding body of work on such networks. To unify the disparate terminology in the large body of recent work, we discuss a general framework for multilayer networks, construct a dictionary of terminology to relate the numerous existing concepts to each other and provide a thorough discussion that compares, contrasts and translates between related notions such as multilayer networks, multiplex networks, interdependent networks, networks of networks and many others. We also survey and discuss existing data sets that can be represented as